Light field integration in SUGRA theories

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We study, in a supersymmetric manifest way, the possibility of having a reliable two-derivative $\mathcal{N} = 1$ Supergravity effective theory in situations where the fields that are mapped out have masses comparable to the supersymmetry breaking scale and the masses of the remaining fields.
We find that in models with two chiral sectors, H and L, described by a Kähler invariant function with schematic dependencies of the form $G = G_H(H, \bar{H}) + G_L(L, \bar{L})$ the superfield equation of motion $\partial_H G = 0$ leads to a reliable two-derivative Supersymmetric description for the L sector upon requiring slowly varying solutions in the H one. The H fields can be charged only under a hidden gauge sector and the dependency of the visible gauge kinetic function on them should be suppressed, the same for the L-field dependency on the hidden gauge kinetic function. In this case the superfield vector equation $\partial_V G = 0$ for the hidden sector is also necessary to get rid of the vector superfields, some of them now massive by gauge symmetry breaking, while mapping out unfixed chiral fields related to the would-be Goldstone directions.
Our results coincide with the naive expectation of promoting to the superspace the F-flatness condition which despite the fact of not being a chiral superfield equation, for these kind of factorizable models, has solutions consistent with chiral superfields.

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I. INTRODUCTION

Supersymmetry (SUSY) and in general Supergravity (SUGRA) not only continues to be the preferred playground for models beyond the Standard Model, but also an ideal framework for dealing with situations where otherwise many calculation would be either impossible or unreliable. However, any constructed model has in mind only a small subset of the entire bunch of fields present in explicit realizations, and these are regarded as encoding all the important dynamics under study. Physically what one has in mind is that the rest of the fields are either decoupled or that their dynamics are negligible. Formally the neglected fields are supposed to be integrated out in such a way that the resulting theory is, at least approximately, SUSY.
Integrating out fields, in a SUSY fashion, in $\mathcal{N} = 1$ SUGRA theories lead recently to some discussion[1–10] settled finally by the work of Brizi, Gomez-Reino and Scrucca[11] where, by requiring an effective two-derivative SUSY description, approximate superfield equations of motion (e.o.m.) were derived, for the fields to be integrate out, together with the estimation of the deviations from the exact effective higher order theory. These can be understood in the light of a low-energy effective theory where higher order terms appear suppressed by the mass of the fields being integrated out and therefore turn out to be subleading. A general result of the work by Brizi et al. is that the gravitational effects to the e.o.m. are automatically negligible once the masses of the integrated fields lie far above the characteristic energies of the effective theory, which include now the SUSY breaking scale, and therefore the leading superfield e.o.m. coincide with the ones of rigid SUSY.
There are, however, scenarios where one might like to get rid of some fields despite no hierarchy is realized. Already in ordinary field theories it is clear that in such a case higher order derivative terms are not longer suppressed, as the kinetic energies in the effective theory are comparable to the masses of the integrated fields. An obvious situation that circumvents this problem is the case where both sectors, the one to be integrated out and the one to be kept, denoted hereafter by $\{H\}$ and $\{L\}$ respectively, are completely decoupled, like in sequestered models.[12–14] For rigid $\mathcal{N} = 1$ SUSY, without vectors fields, this is obtained for Kähler potential and superpotential factorized schematically as follows

$$K = K_H(H) + K_L(L)\,, \quad W = W_H(H) + W_L(L)\,. \quad (1)$$

In SUGRA, however, the theory is described by the generalized Kähler invariant function, $G = K + \ln|W|^2$, so the factorization is not present in G nor in the theory. Moreover, gravitational interactions makes that even if G turns out to have a factorized form, i.e., $G = G_H(H) + G_L(L)$, the Lagrangian has not a fully decoupled structure as can be seen already in the scalar potential,

$$V = e^G\left(G^{I\bar{J}} G_I G_{\bar{J}} - 3\right)\,, \quad (2)$$

with $G^{I\bar{J}} \equiv \left(G_{I\bar{J}}\right)^{-1}$ the inverse scalar manifold metric, the subindex I denoting derivatives respect to the superfield ϕ^I, and everything is evaluated in the lowest

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component of the superfields.
This was first discussed in ref.15 and later on studied in ref.4, 5, and 8. A more detailed study was done by the author in ref.16, where also vector fields where included, but still following a component approach where SUSY is not manifest in the prescription and therefore is not completely clear how to understand the results in a fully SUSY framework. The present letter follows closely the analysis in ref.11, now for the factorizable models, looking for the conditions for a reliable two-derivative SUSY description in the effective theory. We conclude that the naive guess for the chiral superfield e.o.m., namely promoting the F-flatness condition to a field identity at the superspace level, is correct upon regarding slowly varying solutions in the H sector. In presence of gauge symmetries, in order the H sector not to be sourced back, the fields to be integrated out can be charged only under some hidden gauge group whose gauge kinetic function dependency on the L sector should be suppressed. In the same way the gauge kinetic function of the visible sector depends on the H fields in a mild way. Then, again upon having slowly varying solutions, the superfield e.o.m.'s are the superspace promotion of the F-flatness conditions plus the D-flatness ones.
Our results are consistent with the findings of ref.16, working only with the scalar Lagrangian, and particularly relevant in the context of SUSY breaking scenarios, and the related issue of moduli stabilization in Superstring/M-theory, where most of the fields are regarded as SUSY preserving and a detailed description of the SUSY breaking and moduli stabilization is performed only on a tiny subset of fields. In particular the seminal work of Kachru, Kallosh, Linde and Trivedi[17] falls in the kind of scenarios where a hierarchy, dictated by the ratio between the flux and non-perturbative dynamics, is present and therefore the results of Brizi et al. apply. On the other hand, for natural Vacuum Expectation Value (VEV) of the superpotential, i.e., $\langle W\rangle \sim 1$ in Planck units, all fields get important, and of the same order, gravity contributions to the masses and therefore no hierarchy is realized. Low-energy SUSY is still possible if the VEV for the G function is negatively large thanks to the universal factor e^G in the potential. This is what precisely happens in the so called Large volume scenarios (LVS) for type-IIB Superstring compactifications[18,19] where, moreover, the coupling between the Khähler moduli, T, and the SUSY preserving dilaton and complex structure moduli, denoted by U, is described by

$$G_{mix} \sim \frac{\xi(U,\bar{U})}{\mathcal{V}(T,\overline{T})} + \frac{W_{np}(T,U)}{W_{flux}(U)} + h.c. \,, \qquad (3)$$

with ξ a function of the dilaton resulting from α' corrections[20] and $W_{flux} \gg W_{np}$ the flux induced and non-perturbative parts in the superpotential. Thus, for large values of the compact manifold volume, $\mathcal{V}$, the G function realizes an approximate factorizable form and our results apply (for details and numerical examples see ref.16).
We should mention that, although with some broad applicability in moduli stabilization models, our analysis should be recasted for scenarios where higher order operators are relevant and the two-derivative level leads to poor descriptions, like the case of cosmological models of inflation where the background dynamics should be taken into account.[21–25] Then, it is necessary to keep full track of the higher order operators to get insights of the effective SUGRA theory.[26]
The letter is organized as follows: section two is dedicated to review the arguments and results in ref.11 for a two-derivative SUSY low-energy effective action with only chiral superfields and shows how the factorizable models can have also a reliable SUSY effective description. The third section is dedicated to study the case where gauge symmetries enter in the game and a fourth one discusses the gravitational terms and the gauge fixing of the superconformal symmetry. We close with some summary and discussion of the results.

II. TWO-DERIVATIVE SUSY EFFECTIVE THEORIES

The discussion in ref.11 starts by noticing that the usual two-derivative truncation for an effective description of a field theory is not enough when SUSY is required, as higher order terms in the spinor bilinears and auxiliary fields are mapped, by SUSY transformations, to higher order derivative terms. Therefore, a further expansion in spinor bilinears and auxiliary fields should be imposed and the truncation at the two-derivative level is valid only if the missing terms are negligible. At the superfield level this translates to neglect SUSY covariant derivatives in the Kähler potential and superpotential in the effective description. In other words, the solutions to the superfield e.o.m., for the fields that are being mapped out, should correspond either to field configurations where all the SUSY covariant derivatives are negligible, or such that are independent of any non-negligible one.
We work directly with the Kähler invariant function, $G = K + \ln|W|^2$, as this Kähler gauge is usually cleaner in the results and therefore convenient for cases where the superpotential is non vanishing or, as in our case, does not introduce any important scaling by say a tiny VEV. It is also convenient to use the superconformal formalism and compensator technique to write down the action.[27–29] In this setup the off-shell minimal SUGRA supermultiplet is split and one of the two auxiliary fields is now contained in a compensator chiral supermultiplet Φ, required by Weyl symmetry, which later on is gauge fixed in order to recover the actual symmetries of SUGRA. Under this formalism the tensor calculi are almost the same of rigid SUSY, allowing to write down the Lagrangian as an integral over rigid supercoordinates. In our Kähler gauge, for the moment without gauge inter-

actions, the Lagrangian reads:[29,30]

$$\mathcal{L} = -3 \int d^2\theta d^2\bar{\theta} e^{-G/3} \Phi \bar{\Phi} + \int d^2\theta \Phi^3 + h.c. + \cdots\,, \quad (4)$$

the ellipses containing terms implying the graviton, gravitino and the remaining auxiliary field from the SUGRA multiplet, also including couplings with the matter multiplets. For the moment we neglect them in our analysis and comment about the consistency of the procedure at the end.

We regard models with two sectors of chiral fields $\{H^i\}$ and $\{L^\alpha\}$ (notice the distinction in the indices), being the H the ones to be integrated out. The exact superfield e.o.m. for the H^i then reads:

$$-\frac{1}{4}\Phi \overline{\mathcal{D}}^2 \left(G_i e^{-G/3}\bar{\Phi}\right) = 0\,, \quad (5)$$

where we have used the identity $\int d^2\bar{\theta} = -\frac{1}{4}\overline{\mathcal{D}}^2$, $\mathcal{D}$ the SUSY covariant derivative, so to get a derivative superfield equation. Regarding $\Phi \neq 0$ and expanding the previous expression we have,

$$e^{-G/3}\bar{\Phi}\left(G_{i\bar{I}\bar{J}}\overline{\mathcal{D}}\bar{\phi}^{\bar{I}}\overline{\mathcal{D}}\bar{\phi}^{\bar{J}} + G_{i\bar{I}}\overline{\mathcal{D}}^2\bar{\phi}^{\bar{J}}\right) + G_i\overline{\mathcal{D}}^2\left(e^{-G/3}\bar{\Phi}\right) + 2G_{i\bar{I}}\overline{\mathcal{D}}\bar{\phi}^{\bar{I}}\overline{\mathcal{D}}\left(e^{-G/3}\bar{\Phi}\right) = 0\,, \quad (6)$$

where for simplicity in the notation we omit the spinor index in the SUSY covariant derivatives, and the I, J indices run over all superfields H^i and L^α. From the previous arguments, the SUSY two-derivative description is reliable if somehow around the solution to the e.o.m. we can neglect the covariant derivatives.

A possibility, studied by Brizi et al. in ref.11 (see also ref.8), is one where, around the solution, the energy scale associated to the second non-mixed holomorphic derivatives of the superpotential, i.e., W_{ij}, dominates over all others, say the ones associated to the superpotential itself, pure L-sector and mixed derivatives, e.g., $W_{\alpha\beta}$ and $W_{i\alpha}$, the space-time derivatives on the fields and the auxiliary fields VEV's. Then, with a regular behavior in the Kähler potential, the dominating term in eq.(6) is the one proportional to G_i, whose leading part is W_i/W, so the approximate e.o.m. reads:

$$W_i = 0\,, \quad (7)$$

which leads to a two-derivative SUSY description for the L fields as no SUSY covariant derivative is present. In particular the solutions to this e.o.m. are vanishing H auxiliary fields implying no contribution to the SUSY breaking from the H sector at leading order.

Physically the fact that the holomorphic W_{ij} derivatives dominate means that the masses of the H fields, $M_H \sim W_{ij}$, are larger than the remaining energy scales, namely the masses and kinetic energy of the L-sector fields and the SUSY breaking scale. Then, the theory obtained from this leading e.o.m. coincides with the full higher order operator effective theory at first order in an expansion in derivatives, spinor bilinears and auxiliary fields, with the missing terms suppressed by M_H, precisely like in any standard low-energy effective description. At the superfield level any missing operator in the effective G function obtained by the approximate e.o.m. comes suppressed by m_L/M_H, with m_L a characteristic energy scale in the L sector, e.g., the masses.

A second possibility is one where the dynamics ruling both sectors are of the same order and therefore no significant hierarchies appear. Still it might be possible to get rid of one sector if somehow it turns out to be decoupled, although we no longer speak about a low-energy description. A decoupling that leads to a SUSY effective theory, proposed first in ref.15 and studied at the level of the scalar potential in ref.4, 8, and 16, can be summarized in a Kähler invariant function with the following structure

$$G = G_H(H,\bar{H}) + G_L(L,\bar{L}) + \epsilon\, G_{mix}(H,\bar{H},L,\bar{L})\,, \quad (8)$$

with G_H and G_L of the same order of magnitude and ϵ small, parameterizing the coupling between the two sectors. We emphasize, however, that the smallness of the mixing is not necessarily due to a small coupling but rather that around the solutions to the e.o.m. all mixed terms turn out to be small. This form for the G function in the superfield e.o.m. implies that all mixed derivatives, $G_{i\alpha}$ or higher order, are suppressed so that the leading terms are proportional either to covariant derivatives of the H fields or to the G_i. Schematically this is:

$$G_i\overline{\mathcal{D}}^2\left(e^{-G/3}\bar{\Phi}\right) + \mathcal{O}\left(\overline{\mathcal{D}H}, \left(\overline{\mathcal{D}H}\right)^2, \overline{\mathcal{D}}^2\bar{H}\right) = \mathcal{O}(\epsilon)\,. \quad (9)$$

For solutions with an approximate two-derivative SUSY description in the effective theory the second term should be negligible and, thus, the first one must cancel independently. Since in general the covariant SUSY derivatives on the L fields are large, this term vanishes only if we require $G_i = 0$. Due to the factorization at leading order the G_i composite superfield depends only on the H components: its lowest component is the evaluation of $\partial_i G_H$ in the H lowest components, meanwhile the higher components are also combinations of the spinor, spinor derivatives and auxiliary fields. Therefore, a vanishing G_i superfield implies vanishing of the H spinor and auxiliary components, as well of the derivatives of spinor ones, a result consistent with the requirement that the H sector preserve SUSY. The SUSY covariant derivative on the H

superfields also depends exclusively on these fields, but furthermore on the space-time derivatives of the lowest component so it is not completely negligible around field configurations solving null G_i's. Then, we conclude that in order to have negligible covariant derivatives, and a approximate two-derivative SUSY description after mapping out the H sector, the space-time derivatives of the H fields should be small. This, contrary to the usual case where the integrated fields are heavy, is not a trivial condition as the H field excitations have masses and kinetic energy naturally in the same energy scale of the ones implied in the L sector.

Here we can exploit the fact that the solutions for vanishing G_i at leading order do not depend on the L fields and, therefore, any consideration on the leading solution for the H sector can be done disregarding the L field configuration. In other words, even though the fields are light the decoupling forbids, at leading order, the excitation of the H sector fluctuation from the L ones. Thus, we follow the usual consideration of moduli stabilization scenarios, namely to regard slowly varying solutions neglecting the space-time derivatives of the H fields, i.e., solving only the extrema of the potential, without worrying that the dynamics of the fluctuations in the L sector would affect such a situation. Under these circumstances the whole SUSY covariant derivative on the H fields is negligible and the solutions to the e.o.m. coincide at leading order in ϵ with the ones of the superfield equation

$$\partial_i G = 0\,. \tag{10}$$

For us is important the fact that the solutions to eq.(10) lead to a two-derivative description which coincides with the full effective theory at leading order in ϵ, as far the space-time derivatives on the H superfield lowest components are small. On the other hand, as was pointed out in ref.8 and 16, eq.(10) coincides at leading order with $\partial_i G_H = 0$ and therefore the solutions are independent of the L fields, so the dynamics of the L sector can be described by a theory where the H sector is regarded as completely frozen, as usually done in moduli stabilization scenarios.

Being more explicit, if the space-time derivatives of the H fields are also of order ϵ, the exact solution for the H superfields has the following parametric form:

$$H = H_o + \epsilon \tilde{H}(L, \bar{L}, \bar{\Phi}, \overline{\mathcal{D}}\bar{L}, \overline{\mathcal{D}}\bar{\Phi})\,, \tag{11}$$

where H_o is the solution to $\partial_i G_H = 0$ and the remaining encodes the non-constant and non-holomorphic part, which in case of not being suppressed would spoil the two-derivative SUSY description. Plugging back the solution into the Kähler invariant function, we have that the effective theory is described by

$$G_{eff} = G_{H,o} + G_L(L, \bar{L}) + \epsilon G_{mix,o}(L, \bar{L}) + \mathcal{O}(\epsilon^2)\,, \tag{12}$$

with the "not" label indicating evaluation at $H = H_o$. Here it is clear that the theory is described, up to next to leading order in ϵ, by a valid G function with no SUSY covariant derivatives and therefore has a reliable two-derivative description.

Let us close this section by drawing attention to a potential issue on eq.(10), and is the fact that the e.o.m. is not a chiral superfield equation. This prevents us, in general, from using it for the integration of chiral fields.[11] In the case of factorizable models, however, this is avoided as the antiholomorphic components of the equations are trivially consistent with the holomorphic ones, in the sense that both lead to vanishing spinor, spinor derivatives and auxiliary components, which at the same time are consistent with the lowest component of the equation that is nothing but the F-flatness condition. Therefore, the leading part in the solution, eq.(11), is given by the chiral set $H_o = \{h_o, 0, 0\}$.

III. GAUGE INTERACTIONS

The presence of gauge interactions modifies the analysis, first by the inclusion of the vector superfields V^A in $G = G(\phi^I, \bar{\phi}^{\bar{I}}, V^A)$, the index A running over the gauge group generators, and then by their kinetic term,

$$\mathcal{L}_{gau-kin} = \frac{1}{4}\int d\theta^2 f_{AB}(\phi^I)\mathcal{W}^A\cdot\mathcal{W}^B + h.c. \tag{13}$$

with superfield strengths $\mathcal{W}_\alpha = -\frac{1}{4}\overline{\mathcal{D}}^2\left(e^{-V}\mathcal{D}_\alpha e^V\right)$, α spinor indices. In this last term the chiral superfields enter only through the gauge kinetic holomorphic function f_{AB}. We do not consider Fayet-Iliopoulos terms as they seem to be inconsistent with SUGRA.[31]

Without assuming any particular form for f_{AB} the e.o.m. for the H^i superfield, eq.(5), is corrected by

$$\partial_i\mathcal{L} \supset \Phi\left[e^{-G/3}\bar{\Phi}\left(G_{i\bar{I}A}\overline{\mathcal{D}}\bar{\phi}^{\bar{I}} + G_{iAB}\overline{\mathcal{D}}V^B + G_{iA}\overline{\mathcal{D}}\right) + 2G_{iA}\overline{\mathcal{D}}\left(e^{-G/3}\bar{\Phi}\right)\right]\overline{\mathcal{D}}V^A - f_{AB,i}\mathcal{W}^A\cdot\mathcal{W}^B\,. \tag{14}$$

These are automatically subleading in case the H fields develop large masses. Indeed, among others, these are related to the SUSY breaking scale through a D-term breaking.

For factorizable models the SUSY covariant derivatives on the L fields do not appear but, even requiring suppressed space-time derivatives in the H fields, eq.(10) is no longer a solution due to the presence of the SUSY

covariant derivatives on vector superfields and therefore the two-derivative description is not valid neither. Notice that all terms inside the brackets in eq.(14) cancel for neutral, i.e., gauge invariant, H fields as all mixed derivatives of G with the vector fields vanish. This is actually a compulsory requirement if we do not want the fields H to be sourced back through the coupling with the gauge fields.[4,10] In fact, since the energy scale we are interested in is of the order of the H field masses there is no kinematic constraint forbidding such a process. Moreover, the solutions we are looking for quite probably imply non vanishing VEV for the lowest components of the H superfields inducing, if charged, spontaneous gauge symmetry breaking. Therefore, by gauge invariance there would be as many flat directions, related to the would-be Goldstone fields, as broken symmetries, meaning that the dynamics from G_H alone cannot fix the whole H sector as the the set of equations $\{G_i = 0\}$ is no longer linear independent. So for a SUSY integration to proceed a D-flatness condition is also needed, which however will depend, in general, on the non negligible SUSY covariant derivatives of the remaining fields. Requiring a neutral H sector is, still, not enough to have SUSY integration as the coupling from the sigma model in the gauge kinetic function follows an identical analysis to the one of the gauge coupling above. Thus, in order to have a two-derivative SUSY description, at least at leading order, we should deal with neutral H fields which enter suppressed in the gauge kinetic function. In such a case we have again that the leading e.o.m. is given by eq.(10) implying an integration of the H sector with a reliable two-derivative SUSY effective description.

There might be particular situations where the D-term SUSY breaking turns out to be suppressed, for instance in the LVS studied in ref.16, and therefore the back-reaction in the H sector is mild enough to be subleading. Then as far as for the scalar potential is regarded, up to the mass level, no suppression in the gauge kinetic function is needed for a leading SUSY freezing of the H sector.[16] However, this does not imply negligible contributions to the H superfields solutions coming from other components of $\mathcal{W}^{\alpha,A}$, e.g., the field strength and gauginos in the Wess-Zumino gauge, which would induce, in particular, non suppressed higher order derivative terms for the vector fields and higher order fermion bilinear for the gauginos. So contrary to the case studied by Brizi et al. (see also ref.10), where these higher order terms are suppressed by the mass of the H fields and therefore negligible, an approximate two-derivative SUSY effective theory is only realized for suppressed dependencies of the H fields in the gauge kinetic function.

We can allow the H fields to be charged under a hidden gauge sector $\mathcal{G}_H$, such that the whole gauge symmetry is given by $\mathcal{G} = \mathcal{G}_H \otimes \mathcal{G}_L$ and the vector superfields are split as $V^a \in \mathcal{G}_L$ and $V^r \in \mathcal{G}_H$, labeled by lower case letters in the beginning and middle of the alphabet respectively. In this case the e.o.m. for the vector hidden superfields should be also implemented. With no loss of generality in order to be more explicit we show the Abelian case for which the e.o.m. reads:

$$
\begin{aligned}
&G_r e^{-G/3}\bar{\Phi}\Phi \\
&+\frac{1}{8}\left[\mathcal{D}^{\alpha}\left(f_{rp}\overline{\mathcal{D}}^2\mathcal{D}_{\alpha}V^p\right)+\overline{\mathcal{D}}_{\dot{\alpha}}\left(\bar{f}_{rp}\mathcal{D}^2\overline{\mathcal{D}}^{\dot{\alpha}}V^p\right)\right]=0\,,
\end{aligned}
\tag{15}
$$

where α and $\dot{\alpha}$ here stand for the spinor index and we have regarded no kinetic mixing in the gauge sector, i.e., $f_{ar} = 0$.

Then, requiring the H field dependencies of the gauge kinetic function for $\mathcal{G}_L$ to be suppressed, only the SUSY covariant derivatives on the H and $\mathcal{G}_H$ sectors appear in the e.o.m for the H fields. The same should be imposed for the dependency of the $\mathcal{G}_H$ gauge kinetic function on the L fields, otherwise their covariant derivative would appear in the e.o.m. in eq.(15). The implementation of the vector superfield integration corresponding to broken symmetries requires a gauge fixing, being the unitary gauge the one with clearest physical interpretation. However, in practice it is useful to work in a gauge where a chiral superfield, with no vanishing component in the would-be Goldstone direction, is simply fixed to its VEV. Then, as far the SUSY covariant derivatives on the H and V^r superfields are negligible there is a reliable two-derivative SUSY description after the integration of the fields through the set of e.o.m.

$$
G_{\tilde{i}} = 0\,, \quad G_r = 0\,, \tag{16}
$$

where $\tilde{i}$ runs over the chiral fields not affected by the gauge fixing. This set of equations are solved, at next to leading order in ϵ, by vanishing spinor and auxiliary components in both the chiral and vector superfields, and for this last one also the θ^2 and $\bar{\theta}^2$ components and the vector field have null solutions. As before this is not enough to guarantee null covariant derivatives and small space-time derivatives should be imposed by hand in order to have reliable a two-derivative SUSY effective description. Then, if the suppressed dependencies mentioned above are of order ϵ, the effective theory is described by a G function with the same form of eq.(12), remaining to include the V^a in G_L and G_{mix} and the V^r, solutions to $G_r = 0$, in G_H and G_{mix}.

IV. GRAVITATIONAL SECTOR AND GAUGE FIXING

In the previous analysis we disregard the gravitational sector contribution to the action encoded in the ellipses in eq.(4). On the other hand we have shown that the effective theory at leading order in ϵ is described by a theory with superconformal symmetry, namely the one obtained by the G function with the H superfields frozen out. Therefore, since the gravitational terms are univocally dictated by the covariance of the symmetries these terms are also well described by the truncated theory.

However, the dilatation, axial and S symmetries, not being actual symmetries of SUGRA, should be gauge fixed

through a fixing of the compensator in terms of the chiral superfields. The fixing proceeds by requiring a canonical normalization in the gravity sector action, eliminating for example kinetic mixings with the matter sector. Notice that the form of G implies decoupling only between the H and L sector but not with the compensator, thus, it is not automatically clear that the gauge fixing is the same in both descriptions or, in other words, that the SUGRA theory that is obtained upon the gauge fixing coincide at leading order. Writing the compensator components as $\Phi = \phi\{1, \chi_\phi, U\}$ the fixing reads:[32]

$$\phi \equiv e^{G/6}\,, \quad \chi_\phi \equiv \frac{1}{3} G_I \chi^I\,, \tag{17}$$

where the G function and its derivatives are evaluated in the lowest components of the superfields and χ^I are the spinor components of the chiral multiplets. Since around the solution to the e.o.m. for the H fields the terms not appearing in the truncated theory, namely $G_i\chi^i$, are of order ϵ and the functions G and G_α coincide in both theories at next to leading order, the gauge fixing is the same modulo subleading terms.

One of the main targets of the present letter is to clarify the integration of the fields at the superfield level, however, the gauge fixing in eq.(17) cannot be promoted to the superspace as the compensator is a chiral field and therefore cannot depend on the fields in the anti-holomorphic sector contained in G. A variation to the fixing which can be performed directly in the superspace is the one proposed by Cheung et al. in ref.33 that in our Kähler gauge reads:

$$\Phi \equiv e^{Z/3}(1 + \theta^2 U)\,, \tag{18}$$

with Z a chiral superfield given by

$$Z = \langle G \rangle + \langle G_I \rangle \phi^I\,, \tag{19}$$

where the $\langle\rangle$ means the VEV. Again since the VEV's in both descriptions coincide at leading order and the terms not appearing in the truncated description are suppressed, the Z superfields, and therefore the obtained SUGRA theories, match at leading order.

V. DISCUSSION

In this letter we have studied the possibility of having a SUSY two-derivative description for effective theories resulting from the integration of light fields in $\mathcal{N} = 1$ SUGRA. The consistency of a derivative expansion with SUSY transformations requires a parallel expansion in spinor bilinears and auxiliary terms, that at the superfield level is seen as an expansion in the SUSY covariant derivative and a reliable two-derivative effective description is the one where these can be neglected.

Whenever we speak about an effective description we have in mind a region in the field configuration space around particular solutions of the e.o.m. for the fields that have been integrated out, and in our case such solutions should preserve SUSY, approximately, albeit the remaining fields stand at points where SUSY is spontaneously broken. One possibility is that the SUSY preserving sector is heavy enough to present a hierarchy with the SUSY breaking scale such that the back-reaction from the breaking is suppressed.[8,10,11] For Kähler potentials with no singular behavior such a hierarchy is realized if in particular the gravitational effects, e.g., the contribution to the masses, are suppressed, and therefore the leading superfield e.o.m. coincides with the one obtained in rigid SUSY.

On the other hand no hierarchy is necessary if in the Lagrangian the two sectors are decoupled. In a SUSY fashion such a decoupling is described by a factorizable Kähler invariant function G, eq.(8). Although this was previously realized at the level of the scalar Lagrangian,[4,8,16] our analysis shows that the situation can be understood in a fully SUSY framework by working directly in the superspace. We find that the decoupling leads to subleading contributions from the SUSY covariant derivatives on the L sector, despite the fact these can be large, and then slowly varying field configurations solving eq.(10) coincide at leading order, in a derivative and ϵ expansion, with slowly varying solutions of the exact e.o.m., implying negligible SUSY covariant derivatives from the H sector and, therefore, a reliable two-derivative SUSY description. In other words, the implementation of the expansion in spinor, spinor derivatives and auxiliary fields is contained in the e.o.m., with solutions $F^i \sim \chi^i \sim \partial_\mu \chi^i \sim \epsilon$, but the one for the lowest component space-time derivative should be imposed by hand with the confidence that thanks to decoupling this would not be spoiled by the L sector dynamics. This condition is natural first from the SUSY perspective as the derivatives are mapped to spinor and auxiliary components which by the e.o.m. are vanishing, and second from the fact that we are not dealing with a genuine low-energy description where the kinetic energy can be neglected compared with the masses.

Let us emphasize that the restriction about slowly varying solutions is only required on the H sector. This is understood from the fact that the scalar manifold is factorized at leading order and therefore no kinetic mixing is realized, implying that the e.o.m. for the H fields are independent of the L space-time derivatives whether or not these are large. Therefore, when the solutions are plugged back in the Lagrangian, where the only relevant part for the H fields is the potential, no extra terms carrying derivatives on the L sector appear. In other words the space-time derivative expansion for the L sector is not affected at leading order by mapping out the H one and, as a result, we can trust the two-derivative truncation for the L fields, allowing rapidly varying field configurations, as much as we do in the original theory.

The fact that ours is not a low-energy description alerts about the fact that even in case the fields to be integrated out are neutral these can be sourced back by

the vector fields from the coupling in the sigma model ruled by the gauge kinetic function. Thus, even if the D-term SUSY breaking is mild other terms in the covariant derivative of the gauge vector are not suppressed and therefore no reliable SUSY two-derivative description is available. One should, then, require a suppressed dependency of the gauge kinetic function on the H fields and, in this case, all the covariant derivative contributions to the e.o.m. are negligible at leading order such that the solutions are determined by eq.(10). We conclude, therefore, that integration of neutral chiral fields that enter in the Kähler invariant function in a factorizable way and suppressed in the gauge kinetic function leads to a reliable SUSY two-derivative effective description, obtained through the e.o.m. in eq.(10), upon requiring slowly varying solutions in the H sector.
We can allow charged H fields but only under some hidden gauge group in which case the L fields should appear in a suppressed way in the corresponding gauge kinetic function. Possible flat directions resulting from symmetry breaking are handled by integrating out the vector supermultiplets, after a gauge fixing for the broken directions. Therefore, the full set of e.o.m. includes the F-flatness and D-flatness conditions, as superfield identities, all together.

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